

Nathaniel Johnston

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Degrees and Academic Positions

- **Mount Allison University** Sackville, NB
Associate Professor (tenured) 2020 – present
Assistant Professor (tenure-track) 2015 – 2020
- **University of Guelph** Guelph, ON
Adjunct Professor 2016 – 2022
- **Institute for Quantum Computing** Waterloo, ON
Postdoctoral Fellow 2012 – 2015
– Supervisors: John Watrous and Ashwin Nayak
- **University of Guelph** Guelph, ON
Ph.D. Mathematics 2008 – 2012
– Advisor: David W. Kribs
– Thesis: Norms and Cones in the Theory of Quantum Entanglement
– Graduated with a 99.3% average
- **University of Guelph** Guelph, ON
M.Sc. Mathematics 2007 – 2008
– Advisors: John Holbrook and David W. Kribs
– Thesis: Stabilized Distance Measures and Quantum Error Correction
– Graduated with a 98.3% average
- **University of Guelph** Guelph, ON
B.A.H. Mathematics 2003 – 2007
– Graduated with a Major in Mathematics and a Minor in Statistics
– Graduated with Honours, a 98.6% cumulative average, and Dean's Honour List each year

Awards, Grants & Honours

NSERC Discovery Grant	2022 – 2027
Paul Paré Excellence Award	2021
NSERC Discovery Grant	2016 – 2021
Governor General's Academic Gold Medal	2013
NSERC Postdoctoral Fellowship (PDF)	2012 – 2014
Brock Doctoral Scholarship	2008 – 2012
NSERC Canada Graduate Scholarship (CGS D)	2008 – 2011
Ontario Graduate Scholarship (OGS) (declined)	2008
Mathematics Graduate Scholarship	2008
NSERC Canada Graduate Scholarship (CGS M)	2007
Ontario Graduate Scholarship (OGS) (declined)	2007

Governor General's Academic Silver Medal	2007
Mathematics Graduation Prize	2007
College of Physical and Engineering Sciences Graduation Prize	2007
NSERC Undergraduate Student Research Award	2007
Class of 1970 Scholarship	2006
Moffat Mathematics & Statistics Award	2006
NSERC Undergraduate Student Research Award	2006
Dean's Scholarship	2005
Moffat Mathematics & Statistics Award	2005
Dean's Scholarship	2004
Ted Newton Memorial Scholarship	2004
WebCT Conferencing Prize	2004
Board of Governors' Scholarship	2003 – 2006
University of Guelph Entrance Scholarship	2003

Teaching Experience

- **Applied Calculus (MATH 1151)** Mount Allison University
Instructor Fall 2023, Fall 2020
 – Co-taught with Matthew Betti and Peter Lelièvre in Fall 2020
- **Linear Algebra (MATH 2221)** Mount Allison University
Instructor Winter 2023, 2022, 2021, 2020, 2019, 2018, 2017, Fall 2015
- **Calculus II (MATH 1121)** Mount Allison University
Instructor Winter 2022, Winter 2019
- **Modern Algebra II (MATH 4221)** Mount Allison University
Instructor Winter 2022
- **Real Analysis I (MATH 3111)** Mount Allison University
Instructor Fall 2021
- **Number Theory (MATH 3231)** Mount Allison University
Instructor Winter 2021
- **Advanced Linear Algebra (MATH 3221)** Mount Allison University
Instructor Fall 2020, Fall 2017, Winter 2016
 – Also taught as a reading course in Winter 2020
- **Calculus I (MATH 1111)** Mount Allison University
Instructor Fall 2019, Winter 2018, Fall 2016
- **Modern Algebra I (MATH 3211)** Mount Allison University
Instructor Fall 2019, 2016
- **Multivariable Calculus (MATH 2111)** Mount Allison University
Instructor Fall 2015
- **Selected Topics in Quantum Information (QIC 890/891)** University of Waterloo
Lecturer for Module 2: Entanglement Detection Summer 2014
 – Created assignment and module notes, and taught the module (4 lectures)

- **Advanced Calculus I (MATH*2200)** University of Guelph
Sessional Lecturer *Fall 2011*
 – Created assignments, examinations, course notes, and taught the course
- **Set Theory (MATH*2000)** University of Guelph
Sessional Lecturer *Fall 2010*
 – Created assignments, examinations, and taught the course
- **Calculus (MATH*1210)** University of Guelph
Teaching Assistant *Winter 2010*
 – Led weekly labs guiding students through difficult problems
- **Set Theory (MATH*2000)** University of Guelph
Teaching Assistant *Fall 2006, Fall 2007*
 – Led seminars guiding students through difficult problems and reviewing course material
- **Math Help Desk** University of Guelph
Teaching Assistant *Sept. 2005 – Apr. 2011*
 – Provided one-on-one and group tutoring to students in first and second year math courses

Publications (available at www.njohnston.ca/publications)

Books

3. N. Johnston and D. Greene. *Conway's Game of Life: Mathematics and Construction*. Self-published, 2022. doi:10.5281/zenodo.6097284
2. N. Johnston. *Advanced Linear and Matrix Algebra*. Springer International Publishing, 2021.
1. N. Johnston. *Introduction to Linear and Matrix Algebra*. Springer International Publishing, 2021.

Peer-Reviewed Journal Articles

38. J. Holbrook, N. Johnston, and J.-P. Schoch. Real Schur norms and Hadamard matrices. *Linear and Multilinear Algebra*, 2023. doi:10.1080/03081087.2023.2212317
37. N. Johnston and L. Pipes. Bounding real tensor optimizations via the numerical range. *Electronic Journal of Linear Algebra*, 39:289–306, 2023.
36. N. Johnston, B. Lovitz, and A. Vijayaraghavan. Complete hierarchy of linear systems for certifying quantum entanglement of subspaces. *Physical Review A*, 106:062443, 2022.
35. N. Johnston, S. Moein, R. Pereira, and S. Plosker. Birkhoff–James orthogonality in the trace norm, with applications to quantum resource theories. *Electronic Journal of Linear Algebra*, 38:760–776, 2022.
34. N. Johnston, S. Moein, R. Pereira, and S. Plosker. Absolutely k-Incoherent Quantum States and Spectral Inequalities for Factor Width of a Matrix. *Physical Review A*, 106:052417, 2022.
33. N. Johnston and J. Sikora. Completely positive completely positive maps (and a resource theory for non-negativity of quantum amplitudes). *Linear Algebra and its Applications*, 653:395–429, 2022.
32. B. Lovitz and N. Johnston. Entangled subspaces and generic local state discrimination with pre-shared entanglement. *Quantum*, 6:760, 2022.

31. G. Champagne, N. Johnston, M. MacDonald, and L. Pipes. Spectral properties of symmetric quantum states and symmetric entanglement witnesses. *Linear Algebra and its Applications*, 649:273–300, 2022.
30. N. Johnston, B. Lovitz, and D. Puzzuoli. The non-m-positive dimension of a positive linear map. *Quantum* 3:172, 2019.
29. N. Johnston and O. MacLean. Pairwise completely positive matrices and conjugate local diagonal unitary invariant quantum states. *Electronic Journal of Linear Algebra*, 35:156–180, 2019.
28. N. Johnston, C.-K. Li, and S. Plosker. The modified trace distance of coherence is constant on most pure states. *Journal of Physics A: Mathematical and Theoretical*, 51:414010, 2018.
27. N. Johnston, C.-K. Li, S. Plosker, Y.-T. Poon, and B. Regula. Evaluating the robustness of k -coherence and k -entanglement. *Physical Review A*, 98:022328, 2018.
26. N. Johnston and E. Patterson. The inverse eigenvalue problem for entanglement witnesses. *Linear Algebra and its Applications*, 550:1–27, 2018.
25. N. Johnston, S. Kirkland, S. Plosker, R. Storey, and X. Zhang. Perfect quantum state transfer using Hadamard-diagonalizable graphs. *Linear Algebra and its Applications*, 531:375–398, 2017.
24. J. Chen, S. Grogan, N. Johnston, C.-K. Li, and S. Plosker. Quantifying the coherence of pure quantum states. *Physical Review A*, 94:042313, 2016.
23. N. Johnston, R. Mittal, V. Russo, and J. Watrous. Extended nonlocal games and monogamy-of-entanglement games. *Proceedings of the Royal Society A*, 472, 2016. DOI: 10.1098/rspa.2016.0003
22. C. Napoli, T. R. Bromley, M. Cianciaruso, M. Piani, N. Johnston, and G. Adesso. Robustness of coherence: An operational and observable measure of quantum coherence, *Physical Review Letters*, 116:150502, 2016.
– Selected as an Editors’ Suggestion.
21. M. Piani, M. Cianciaruso, T. R. Bromley, C. Napoli, N. Johnston, and G. Adesso. Robustness of asymmetry and coherence of quantum states. *Physical Review A*, 93:042107, 2016.
– Selected as an Editors’ Suggestion.
20. N. Johnston and D. W. Kribs. Duality of entanglement norms. *Houston Journal of Mathematics*, 41(3):831–847, 2015.
19. S. Bandyopadhyay, A. Cosentino, N. Johnston, V. Russo, J. Watrous, and N. Yu. Limitations on separable measurements by convex optimization. *IEEE Transactions on Information Theory*, 61(6):3593–3604, 2015.
18. S. Arunachalam, N. Johnston, and V. Russo. Is absolute separability determined by the partial transpose? *Quantum Information & Computation*, 15(7 & 8):694–720, 2015.
17. J. Chen and N. Johnston. The minimum size of unextendible product bases in the bipartite case (and some multipartite cases). *Communications in Mathematical Physics*, 333(1):351–365, 2015.
16. N. Johnston. The structure of qubit unextendible product bases. *Journal of Physics A: Mathematical and Theoretical*, 47:424034, 2014.
15. G. Gutoski and N. Johnston. Process tomography for unitary quantum channels. *Journal of Mathematical Physics*, 55:032201, 2014.

14. N. Johnston. Separability from spectrum for qubit–qudit states. *Physical Review A*, 88:062330, 2013.
13. J. Chen, H. Dawkins, Z. Ji, N. Johnston, D. W. Kribs, F. Shultz, and B. Zeng. Uniqueness of quantum states compatible with given measurement results. *Physical Review A*, 88:012109, 2013.
12. N. Johnston. Non-positive partial transpose subspaces can be as large as any entangled subspace. *Physical Review A*, 87:064302, 2013.
11. N. Johnston. Non-uniqueness of minimal superpermutations. *Discrete Mathematics*, 313:1553–1557, 2013.
 - I was interviewed about this problem for an article in [Quanta Magazine](#).
10. N. Johnston, L. Skowronek, and E. Størmer. Generation of mapping cones from small sets. *Linear Algebra and Its Applications*, 438:3062–3075, 2013.
9. N. Johnston and E. Størmer. Mapping cones are operator systems. *Bulletin of the London Mathematical Society*, 44:738–748, 2012.
8. N. Johnston and D. W. Kribs. Quantum gate fidelity in terms of Choi matrices. *Journal of Physics A: Mathematical and Theoretical*, 44:495303, 2011.
7. N. Johnston. Characterizing operations preserving separability measures via linear preserver problems. *Linear and Multilinear Algebra*, 59:1171–1187, 2011.
6. N. Johnston, D. W. Kribs, V. I. Paulsen, and R. Pereira. Minimal and maximal operator spaces and operator systems in entanglement theory. *Journal of Functional Analysis*, 260:2407–2423, 2011.
5. N. Johnston and D. W. Kribs. A family of norms with applications in quantum information theory II. *Quantum Information & Computation*, 11:104–123, 2011.
4. N. Johnston and D. W. Kribs. Generalized multiplicative domains and quantum error correction. *Proceedings of the American Mathematical Society*, 139:627–639, 2011.
3. N. Johnston and D. W. Kribs. A family of norms with applications in quantum information theory. *Journal of Mathematical Physics*, 51:082202, 2010.
 - Selected for the Virtual Journal of Quantum Information.
2. M.-D. Choi, N. Johnston, and D. W. Kribs. The multiplicative domain in quantum error correction. *Journal of Physics A: Mathematical and Theoretical*, 42:245303, 2009.
1. N. Johnston, D. W. Kribs, and V. I. Paulsen. Computing stabilized norms for quantum operations. *Quantum Information & Computation*, 9:16–35, 2009.

Conference Proceedings

5. N. Johnston. The minimum size of qubit unextendible product bases. In *Proceedings of the 8th Conference on the Theory of Quantum Computation, Communication and Cryptography (TQC)*, 2013. doi:10.4230/LIPIcs.TQC.2013.93
4. N. Johnston. Norm duality and the cross norm criteria for quantum entanglement. *Linear and Multilinear Algebra (Proceedings of the 11th Workshop on Numerical Ranges and Numerical Radii)*, 2013. doi:10.1080/03081087.2012.753595

3. N. Johnston and D. W. Kribs. A family of norms with applications in entanglement theory. In *Proceedings of the 2011 ICO International Conference on Information Photonics (IP)*, 2011. doi:10.1109/ICO-IP.2011.5953727
2. N. Johnston and D. W. Kribs. Schmidt operator norms and entanglement theory. In *Fourth International Conference on Quantum, Nano and Micro Technologies*, pages 92–95, 2010.
– Selected as one of the best papers of the conference.
1. N. Johnston, D. W. Kribs, and C.-W. Teng. An operator algebraic formulation of the stabilizer formalism for quantum error correction. *Acta Applicandae Mathematicae*, 108:687–696, 2009.

Book Chapters

2. N. Johnston. Some Beautiful and Difficult Questions about Cellular Automata. In *Designing Beauty: The Art of Cellular Automata*, A. Adamatzky and G. J. Martinez (eds.), Springer International Publishing, pages 59–63, 2016.
1. N. Johnston. The B36/S125 “ 2×2 ” Life-like cellular automaton. In *Game of Life Cellular Automata*, A. Adamatzky (ed.), Springer-UK, pages 99–114, 2010.

Unpublished Papers

7. N. Johnston, V. Russo, and J. Sikora. *Tight bounds for antidistinguishability and circulant sets of pure quantum states*. E-print: [arXiv:2311.17047](https://arxiv.org/abs/2311.17047) [quant-ph], 2023.
6. N. Johnston, B. Lovitz, and A. Vijayaraghavan. *A hierarchy of eigencomputations for polynomial optimization on the sphere*. E-print: [arXiv:2310.17827](https://arxiv.org/abs/2310.17827) [math.OC], 2023.
5. N. Johnston and S. Plosker. *Laplacian $\{-1, 0, 1\}$ - and $\{-1, 1\}$ -diagonalizable graphs*. E-print: [arXiv:2308.15611](https://arxiv.org/abs/2308.15611) [math.CO], 2023.
4. R. Houston, A. P. Goucher, and N. Johnston. *A New Formula for the Determinant and Bounds on Its Tensor and Waring Ranks*. E-print: [arXiv:2301.06586](https://arxiv.org/abs/2301.06586) [math.CO], 2023.
3. N. Johnston, B. Lovitz, and A. Vijayaraghavan. *Computing linear sections of varieties: quantum entanglement, tensor decompositions and beyond*. E-print: [arXiv:2212.03851](https://arxiv.org/abs/2212.03851) [quant-ph], 2022.
2. N. Johnston. *The complexity of the puzzles of “Final Fantasy XIII-2”*. E-print: [arXiv:1203.1633](https://arxiv.org/abs/1203.1633) [cs.CC], 2012.
1. N. Johnston. *Partially entanglement breaking maps and right CP-invariant cones*. Unpublished notes, 2008.

Presentations

- **Laplacian $\{-1, 0, 1\}$ - and $\{-1, 1\}$ -diagonalizable graphs**
Canadian Mathematical Society Winter 2023 Meeting (Montréal) December 2023
- **Antidistinguishability and k -Incoherence**
Canadian Mathematical Society Summer 2023 Meeting (U. of Ottawa) June 2023
Theory Canada 15 (Mount Allison University) June 2023

- **A New Formula for the Determinant and Bounds on Its Tensor Rank**
TATERS Math Seminar (Boise State University, virtual) April 2023
Joint Mathematics Meetings 2023 January 2023
- **Completely Positive Completely Positive Maps**
Canadian Mathematical Society Summer 2021 Meeting (virtual) June 2021
49th Canadian Operator Symposium (virtual) June 2021
2021 Western Canada Linear Algebra Meeting (virtual) May 2021
- **Pairwise Completely Positive Matrices**
Geometry and Matrix Analysis seminar (U. of Western Ontario) August 2022
2019 Meeting of the International Linear Algebra Society (Brazil) July 2019
Canadian Mathematical Society Summer 2019 Meeting (Regina) June 2019
- **The Minimal Superpermutation Problem**
Dalhousie Mathematics Colloquium (Halifax) December 2018
- **The Absolute Separability Problem in Quantum Information Theory**
Physics Seminar (U. de Moncton) November 2018
Quantum Information & Geometric Statistics Seminar (U. of Guelph) November 2018
5th Int. Conference on Matrix Analysis and Applications (Florida) December 2015
Workshop on Quantum Marginals and Numerical Ranges (Guelph) August 2015
- **What About 3D Matrices?**
Mount Allison Math & CS Society (Sackville) October 2018
- **Hadamard-Diagonalizable Graphs with Perfect State Transfer**
Algebraic Graph Theory & Quantum Walks (Waterloo) April 2018
- **The Spectra Arising from Positive Linear Maps**
Canadian Mathematical Society Summer 2018 Meeting (Fredericton) June 2018
Canadian Mathematical Society Winter 2017 Meeting (Waterloo) December 2017
2017 Workshop on Operator Systems in Quantum Information (U. of Guelph) August 2017
- **Quantum Coherence and Quantum Entanglement**
2017 Meeting of the International Linear Algebra Society (Iowa) July 2017
 – Selected as an LAA Early Career Speaker
- **Qubit Unextendible Product Bases and Graph Colourings**
2017 Prairie Discrete Math Workshop (Saskatchewan) June 2017
2014 SIAM Conference on Discrete Mathematics (Minneapolis) June 2014
8th Conference on Theory of Quantum Computation (U. of Guelph) May 2013
- **Twirling States to Simplify Separability**
Workshop on Representation Theory in Quantum Information (Guelph) August 2016
- **Some Linear Algebra Questions Arising from Quantum Coherence**
Quantum Information & Geometric Statistics Seminar (U. of Guelph) June 2016
2016 Western Canada Linear Algebra Meeting (Winnipeg) May 2016
- **Preservers of UPBs and Local Distinguishability of Quantum States**
Canadian Mathematical Society Summer 2014 Meeting (Winnipeg) June 2014
- **The Separability Problem and its Variants in Entanglement Theory**
Math–Physics Colloquium (U. of New Brunswick) November 2015
Quantum Information & Geometric Statistics Seminar (U. of Guelph) April 2015
Mathematics Colloquium (U. of Louisiana) March 2014
Analysis Seminar (U. of Western Ontario) February 2014
- **Separability from Spectrum for Qubit–Qudit States**
Canadian Mathematical Society Winter 2013 Meeting (Ottawa) December 2013

- **Process Tomography for Unitary Quantum Channels**
Quantum Information & Geometric Statistics Seminar (U. of Guelph) *September 2013*
- **Unextendible Product Bases as Mixed Integer Programs**
2013 Mixed Integer Programming Workshop (Poster – Wisconsin) *June 2013*
- **On the Minimum Size of Unextendible Product Bases**
18th Conference of the International Linear Algebra Society (Rhode Island) *June 2013*
- **Non-Uniqueness of Minimal Superpermutations**
Ottawa–Carleton Discrete Mathematics Days (Ottawa) *May 2013*
- **Uniqueness of Quantum States Compatible with Measurement Results**
Workshop on Mathematical Methods of Quantum Tomography (Poster – Toronto) *February 2013*
Tuesday Theory Lunch (IQC, Waterloo) *January 2013*
- **The NPPT Bound Entanglement Problem**
Summer Research Workshop on Quantum Information Science (China) *July 2012*
- **Right CP-Invariant Cones of Superoperators**
7th Workshop on Matrices and Operators (China) *July 2012*
- **Duality of Entanglement Norms**
11th Workshop on Numerical Ranges and Numerical Radii (Taiwan) *July 2012*
- **The Quantum Separability Problem**
Quantum Information & Geometric Statistics Seminar (U. of Guelph) *July 2012*
- **Isometries of Locally Unitarily Invariant Norms**
International Conference on Mathematics and Statistics (Memphis) *May 2012*
- **Complete Positivity and CP-Invariance in Q.I.T.**
Canadian Mathematical Society Winter 2011 Meeting (Toronto) *December 2011*
Quantum Information & Geometric Statistics Seminar (U. of Guelph) *November 2011*
- **Quantum Gate Fidelity in Terms of Choi Matrices**
Tuesday Theory Lunch (IQC, Waterloo) *March 2011*
Quantum Information & Geometric Statistics Seminar (U. of Guelph) *March 2011*
- **Applications of a Family of Norms in Entanglement Theory**
Institute for Quantum Information Science Seminar (U. of Calgary) *February 2011*
- **Minimal and Maximal Operator Spaces and Operator Systems**
14th Workshop on Quantum Information Processing (Poster – Singapore) *January 2011*
Quantum Information & Geometric Statistics Seminar (U. of Guelph) *November 2010*
- **Linear Preserver Problems in Quantum Information Theory**
Quantum Information & Geometric Statistics Seminar (U. of Guelph) *October 2010*
- **Schmidt Norms for Quantum States**
Quantum Computation & Information Group Seminar (U. of Bristol) *May 2010*
Tuesday Theory Lunch (IQC, Waterloo) *December 2009*
Quantum Information & Geometric Statistics Seminar (U. of Guelph) *November 2009*
- **The Multiplicative Domain in Quantum Error Correction**
Canadian Quantum Information Student Conference (Toronto) *August 2009*
4th Workshop, TQC 2009 (Poster – Waterloo) *May 2009*
Canadian Mathematical Society Winter 2008 Meeting (Ottawa) *December 2008*
Quantum Information & Geometric Statistics Seminar (U. of Guelph) *July 2008*
- **Completely Bounded Norms in Quantum Information**
Quantum Information & Geometric Statistics Seminar (U. of Guelph) *August 2007*

Student and Postdoc Supervision

- Shirin Moein (Jan. 2021 – Jan. 2023, postdoc, co-supervised with Sarah Plosker and Rajesh Pereira)
- Everett Patterson (Sept. 2019 – Apr. 2020, undergraduate thesis)
- Olivia MacLean (Sept. 2018 – Apr. 2019, undergraduate thesis)

Academic Service and Professional Activities

- Co-organized sessions/workshops at the following scientific meetings:
 - Canadian Mathematical Society Winter 2022 Meeting (Toronto)
 - Canadian Mathematical Society Summer 2019 Meeting (Regina)
 - Canadian Mathematical Society Summer 2018 Meeting (Fredericton)
 - Canadian Mathematical Society Winter 2016 Meeting (Niagara Falls)
- Refereed papers for the following journals and conferences:
 - Annals of Physics
 - Annales Henri Poincaré
 - Communications in Mathematical Physics
 - Discrete Mathematics
 - Electronic Journal of Linear Algebra
 - The European Physical Journal D
 - IEEE Transactions on Information Theory
 - International Journal of Quantum Information
 - Journal of Mathematical Analysis and Applications
 - Journal of Mathematical Physics
 - Journal of Operator Theory
 - Journal of Physics A: Mathematical and Theoretical
 - Journal of the Korean Mathematical Society
 - Linear Algebra and its Applications
 - Linear and Multilinear Algebra
 - New Journal of Physics
 - npj Quantum Information
 - Physica Scripta
 - Physical Review A
 - Physical Review Letters
 - PLOS ONE
 - Pramana Journal of Physics
 - Proceedings of the American Mathematical Society

- Quantum
- Quantum Information & Computation
- Quantum Information Processing
- Scientific Reports
- SciPost Physics
- SIAM Journal on Matrix Analysis and Applications
- Workshop on Quantum Information Processing (subreviewer for the 2013, 2019, 2021, 2022, and 2024 workshops)
- Mount Allison representative on the [Science Atlantic Math & Stats Committee](#)
July 2018 – present
- Designed the [January 2015 cover](#) of the College Mathematics Journal
- Editor-in-Chief of the [On-Line Encyclopedia of Integer Sequences](#).
April 2011 – present
- Creator of [QETLAB](#), a MATLAB package for exploring quantum entanglement.
- Creator and maintainer of [ConwayLife.com](#), a widely-used website containing over 2,000 articles about cellular automata.

Technical Skills

- Markup Languages
 - CSS, \LaTeX , (X)HTML
- Programming Languages
 - C, Java, Javascript, PHP, Python, SQL
- Specialized Software
 - Maple, MATLAB, MediaWiki, MySQL